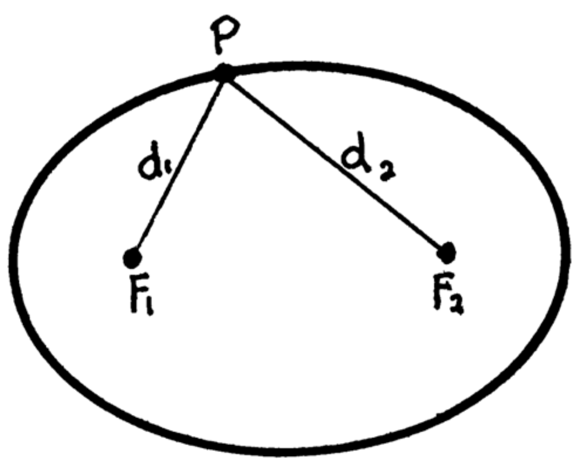
Ellipses

An ellipse is a plane curve that results from the intersection of a cone by a plane in a way that produces a closed curve. Circles are special cases of ellipses, obtained when the cutting plane is perpendicular to the axis. An ellipse is also the locus of all points of the plane whose distances to two fixed points add to the same constant.

What is an ellipse?

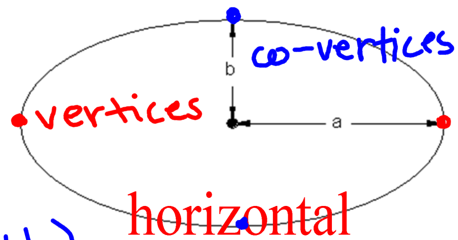
- An ellipse is a set of points in a plane, the sum of whose distances from two fixed points is constant.
- The fixed points are called foci.



$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

V: $(h+a, k)$
$(h-a, k)$

Co-V: $(h, k+b)$
$(h, k-b)$

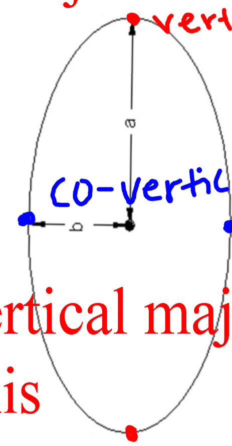


horizontal major axis

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Co-V: $(h+b, k)$
$(h-b, k)$

V: $(h, k+a)$
$(h, k-a)$



vertical major axis

Finding the foci

$c^2 = a^2 - b^2$

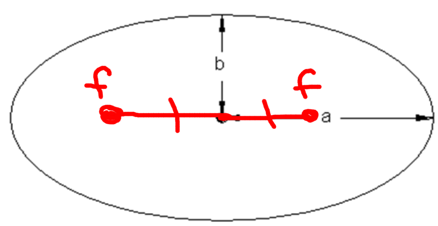
$c = \sqrt{a^2-b^2}$

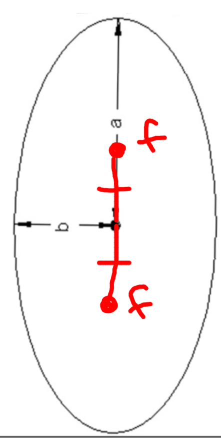
the foci are:

(h+c, k) and (h-c, k)

or

(h, k+c), (h, k-c)





Find the foci, vertices, and co-vertices of each ellipse.

$$\frac{x^2}{121} + \frac{y^2}{144} = 1$$

$b = 11$ $a = 12$

$(0, 0)$

$V: (0, 12)$ $(0, -12)$

$c^2 = 23$
$c = \sqrt{23}$

$CV: (11, 0)$ $(-11, 0)$

$F: (0, \sqrt{23})$ $(0, -\sqrt{23})$

$$\frac{x^2}{100} + \frac{y^2}{196} = 1$$

$b = 10$ $a = 14$

$V: (0, 14)$ $(0, -14)$

$CV: (0, 10)$ $(0, -10)$

$c^2 = 96$
$c = \sqrt{96}$ $<^{8}_{12}$ $<^{2}_{4}$, $<^{4}_{3}$ $= 4\sqrt{6}$

$F: (0, 4\sqrt{6})$ $(0, -4\sqrt{6})$

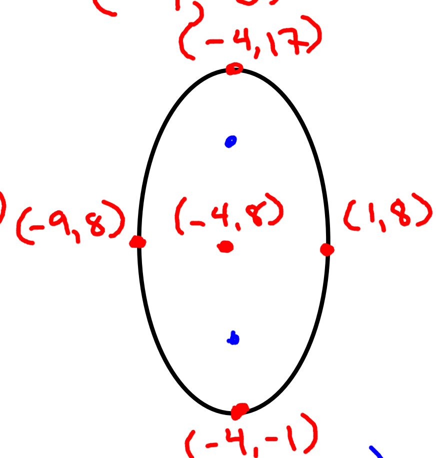
$$\frac{(x + 4)^2}{25} + \frac{(y - 8)^2}{81} = 1$$

$b = 5$ $a = 9$

$(-4, 8)$

$CV: (1, 8)$ $(-9, 8)$

$V: (-4, 17)$ $(-4, -1)$



$c^2 = a^2 - b^2$
$c^2 = 81 - 25$
$c^2 = 56$
$c = \sqrt{56} = 2\sqrt{14}$

56: 7, 8
8: 2, 4
4: 2, 2

$F: (-4, 8 + 2\sqrt{14})$ $(-4, 8 - 2\sqrt{14})$

$$\frac{(x+2)^2}{49} + \frac{(y-1)^2}{16} = 1$$

$(-2, 1)$

$a = 7$ $b = 4$

V: $(5, 1)$ CV: $(-2, 5)$

$(-9, 1)$ $(-2, -3)$

$c^2 = 49 - 16$

$c^2 = 33$

$c = \sqrt{33}$

F: $(-2 + \sqrt{33}, 1)$

$(-2 - \sqrt{33}, 1)$